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ABSTRACT

A natural circulation loop with water as the circulating fluid was studied for a range of operation covering two-phase flow. Periodic oscillations of the flow rate and fluid temperature occur even with constant heatinput and constant cooling water properties for the heat exchangers. Several conclusions concerning the stability of operation are given. Use is made of the theoretical analysis of an open-ended system, and an analogue computer. For use in a more detailed numerical analysis, the equation of motion, the continuity equation, and the energy equation are presented for a transient two-phase flow model.

The Oscillatory Behavior of a Two-Phase Natural Circulation Loop

by

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Introduction

A program of study of the transient operation of natural circulation loops has been underway at the University of Minnesota (1), and this paper is concerned with the oscillatory behavior of a two-phase natural circulation loop. These studies are of interest for the emergency cooling of nuclear reactors and in the design of boiling water reactors. The literature survey pertaining to the transient operation of a natural circulation loop is given by Alstad, Isbin, Amundson and Silvers (1), and a survey on two-phase flow is given by Isbin, Moen and Mosher (2).

Experimental Loop

Figure 1 is a schematic diagram of the natural convection loop which was studied. The loop was constructed primarily of 16 gage, 1 inch O.D. (0.872 inch I.D.) hard drawn brass tubing. The major features of the loop are described in reference (1). During a natural circulation run, the flowrators were by-passed and only the electromagnetic flowmeter was used. Normally the surge tank,

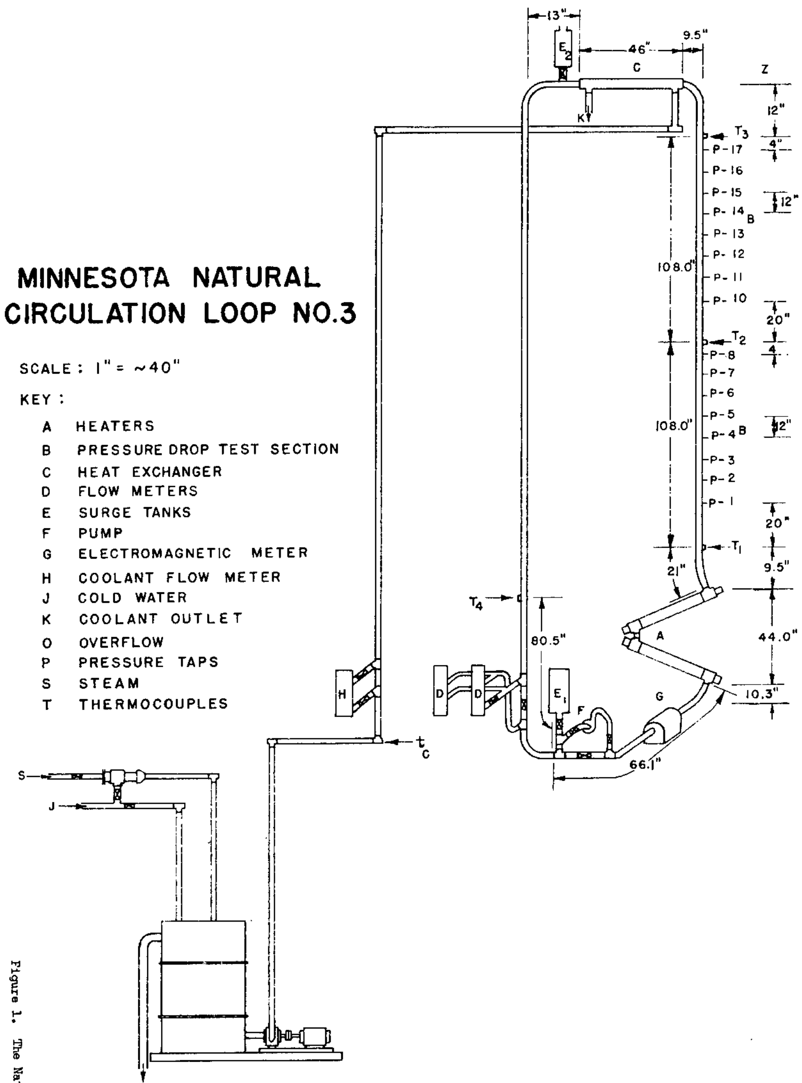

Figure 1. The Natural Convection Loop.

E_1, was not used for two-phase natural circulation runs. For those runs in which the pressure at one point in the loop was held constant, the gate valve between the surge tank, E_2, and the loop was opened; for the constant volume run, the gate valve was closed. A heater and pump were installed to maintain the cooling water supply at 5 gpm and at 130°F.

Theoretical Analysis

General Equations The continuity equation, the equation of motion and the energy equation for a viscous fluid flowing in a region of general geometry were formulated for a phase having continuous properties. A similar set of equations were derived for flow across a surface of discontinuity. The combination of these two sets of equations permits one to write the equations for the two-phase flow. An annular flow model was selected to illustrate some of the essential properties of two-phase flow. The model is sufficiently simple to permit attempting a numerical solution. Each phase is assumed to flow through a well defined cross-sectional area with a uniform velocity with the reservation that the liquid velocity at the wall must equal zero. Further, the steam and water phases are assumed to be in equilibrium; that is, p and T correspond to the saturation pressure and temperature.

For these conditions, the continuity equation may be written as

$$A \frac{\partial \rho}{\partial t} = - \frac{\partial W}{\partial x} \quad (1)$$

The equation of motion for upward flow through a pipe of constant diameter becomes

$$\frac{1}{g_c} \frac{\partial W}{\partial t} + \frac{1}{g_c} \frac{\partial (W\phi_1 V)}{\partial x} = - A \frac{\partial p}{\partial x} - A \frac{g}{g_c} \rho z' - A F_f \quad (2)$$

Finally, the energy equation is

$$A \frac{\partial (\rho H)}{\partial t} + \frac{\partial (W\phi_2 H)}{\partial x} - \frac{A}{J} \frac{\partial p}{\partial t} = Q' + \frac{\partial}{\partial x} [(k_w A_w + k_s A_s) \frac{\partial T}{\partial x}] \quad (3)$$

where $2\pi r_p k_w \left(\frac{\partial T}{\partial r}\right)_{r_p} = Q'$ = rate at which heat is conducted through the pipe.

The terms ϕ_1 and ϕ_2 represent the ratio of the true rate of transport to the rate of transport of the mean flow for momentum and energy respectively. If both phases have the same linear velocity, ϕ_1 and ϕ_2 are unity.

The three functions ϕ_1, ϕ_2 and F_f were determined experimentally from steady state data. It was found that ϕ_1 and ϕ_2 could be correlated as functions of ρ alone, and that

$$F_f = a(\rho) W^{1.79} \quad (4)$$

where $a(\rho)$ is a function of ρ. Figures 2, 3 and 4 illustrate the variation of ϕ_1, ϕ_2 and $a(\rho)$ with ρ.

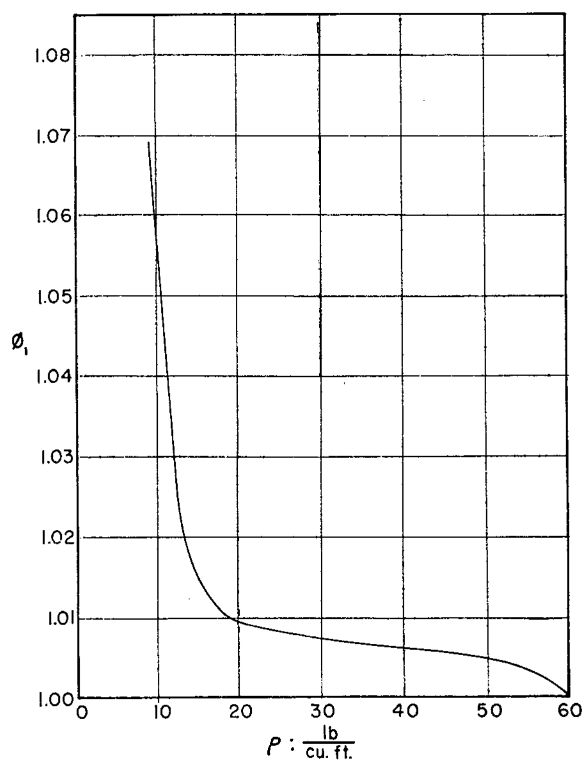

Figure 2. The Momentum Transport Coefficient, ϕ_1.

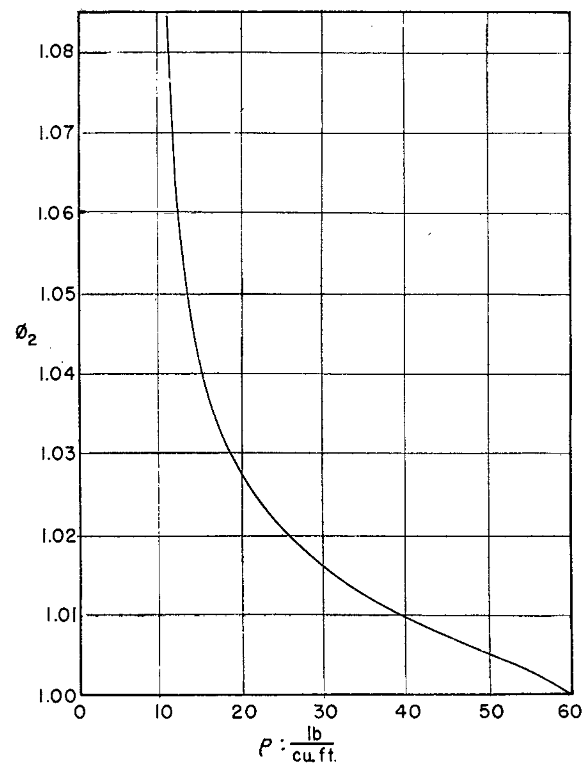

Figure 3. The Energy Transport Coefficient, ϕ_2.

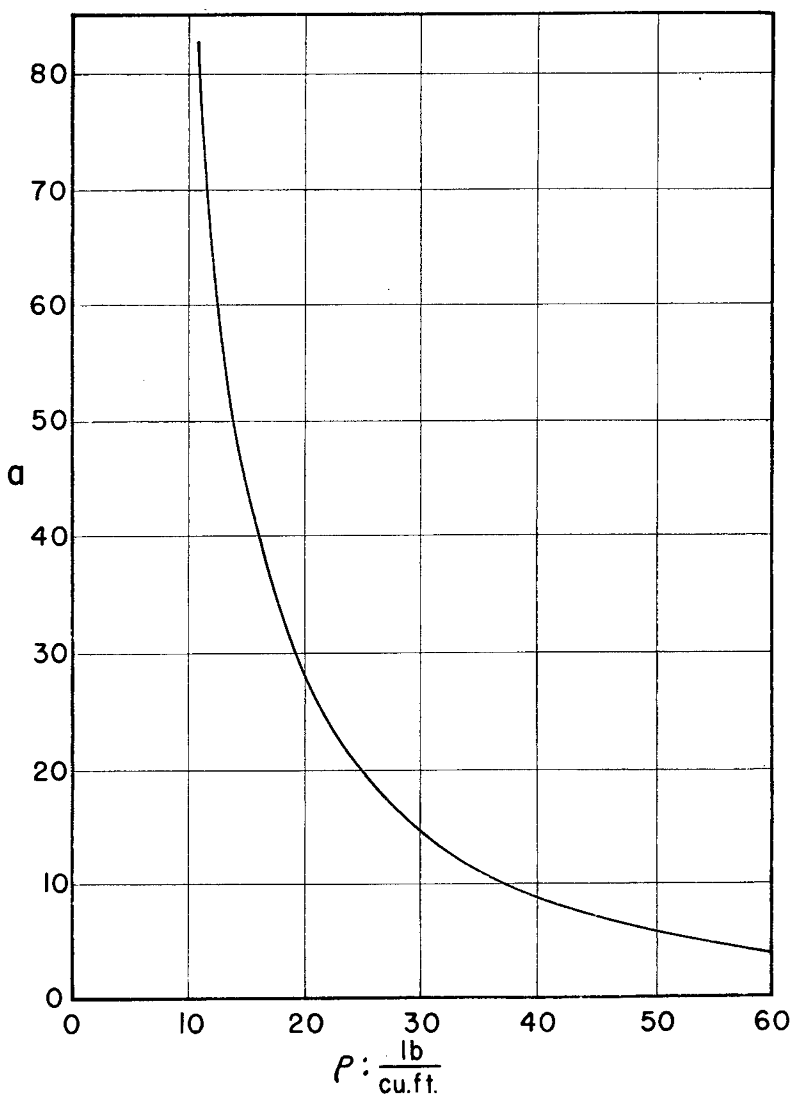

Figure 4. The Friction Coefficient, a.

Equations (1) through (4) have been applied to a natural circulation loop in the form of finite difference equations. The discussion of these equations is not included in this paper for the numerical calculations using the SEAC (National Bureau of Standard's digital computer, Standard Eastern Automatic Computer) have not been successfully completed.

Stability Analyses

An insight on the factors which determine the stability of a natural convection system is gained through the analysis of an open-ended loop, such as shown in Figure 5. The fluid entering the heater always has the constant temperature T_{oi}, and the velocity of the stream is fixed by the density difference between the hot and cold leg. For any constant heat input, one may define a state of equilibrium in which the difference in weight of the two legs is just equal to the frictional resistance to flow. Under certain conditions this system may be unstable; that is a small deviation from the equilibrium temperature distribution or the equilibrium velocity may be propagated in space or time with increasing amplitude. For example, if a velocity perturbation of the form, $\epsilon_v \sin\omega t$, is present, under what conditions could the temperature perturbation which is generated cause the driving force that sustains the velocity

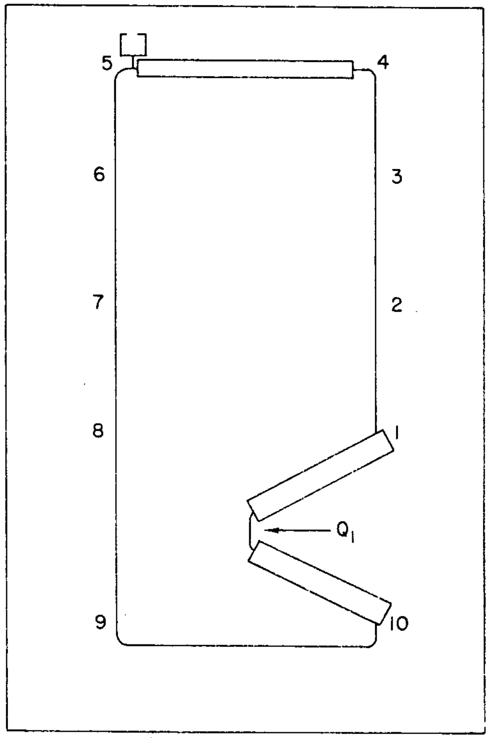

Figure 6. Subdivisions for the Analogue Computer Problem.

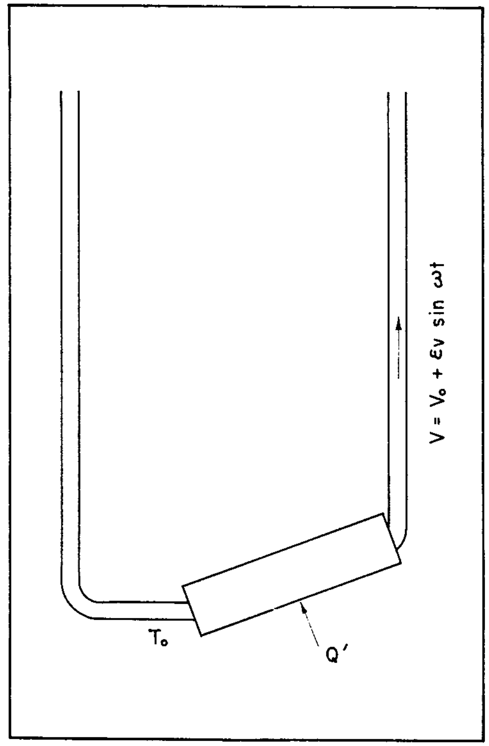

Figure 5. The Stability Model

disturbance? The problem has been treated analytically for a one-phase fluid (3), and the following conclusions were obtained:

a) If an oscillatory flow rate is to be possible, the force cannot be generated in the heater; it must be generated in the vertical riser.

b) The product of the coefficient of expansion of the fluid and the vertical height of the riser must exceed a certain value (defined by an analytical expression) if the velocity perturbation is to be sustained.

c) The period of oscillation will be approximately equal to the residence time of the fluid in the heater and the vertical riser.

In order to predict the period for a closed loop, a problem was solved on a Reeves Electronic Analogue Computer. The number of non-linear terms was limited to 16, and the model used was necessarily a simple one. For such a problem, the period of the oscillations should be meaningful even though the wave shape is not correct, if the period is a function of only the geometry and the mean velocity. The computer could handle only ten subdivisions, with the driving force expressed as a linear function of all ten enthalpies, and the frictional force as a quadratic function of the velocity. The number

of available summing amplifiers limited the problem to the case in which boiling occurs only at the top subdivision of the vertical riser. The equations solved were of the following form:

$$\frac{dH_n}{dt} = -\frac{V}{(A\Delta x)_n}(H_n - H_{n-1}) + \frac{Q_n}{(A\Delta x)_n \bar{\rho} C_p} \qquad (5)$$

$$\frac{dV}{dt} = \frac{-\Sigma\rho_n z'_n \Delta x_n - F}{\frac{\bar{\rho}_n}{g_c}\Sigma\frac{\Delta x_n}{A_n}} \qquad (6)$$

The loop subdivisions are given in Figure 6. For section 1, $Q_n = Q_1$, for section 5, $Q_5 = -bV(\frac{H_5 + H_4}{2} - h_c)$ where the cooler heat transfer coefficient is taken to be a linear function of velocity, and all remaining Q's are taken equal to zero. The density at point n is written as

$$\rho_n = \rho_{n_o} + \alpha_n (H_n - H_{n_o}) \qquad (7)$$

and the mean density of the nth subinterval is set equal to the average of the densities at n and n-1.

A stable solution was found if the coefficient of expansion of water is used for all values of α_n. An oscillatory solution was obtained, Figure 7, if vaporization to a few per cent quality ~~were~~ assumed in the top section of the vertical riser ($\alpha_4 = -9.500$, all other α's = -0.024 lb/cu.ft.°F).

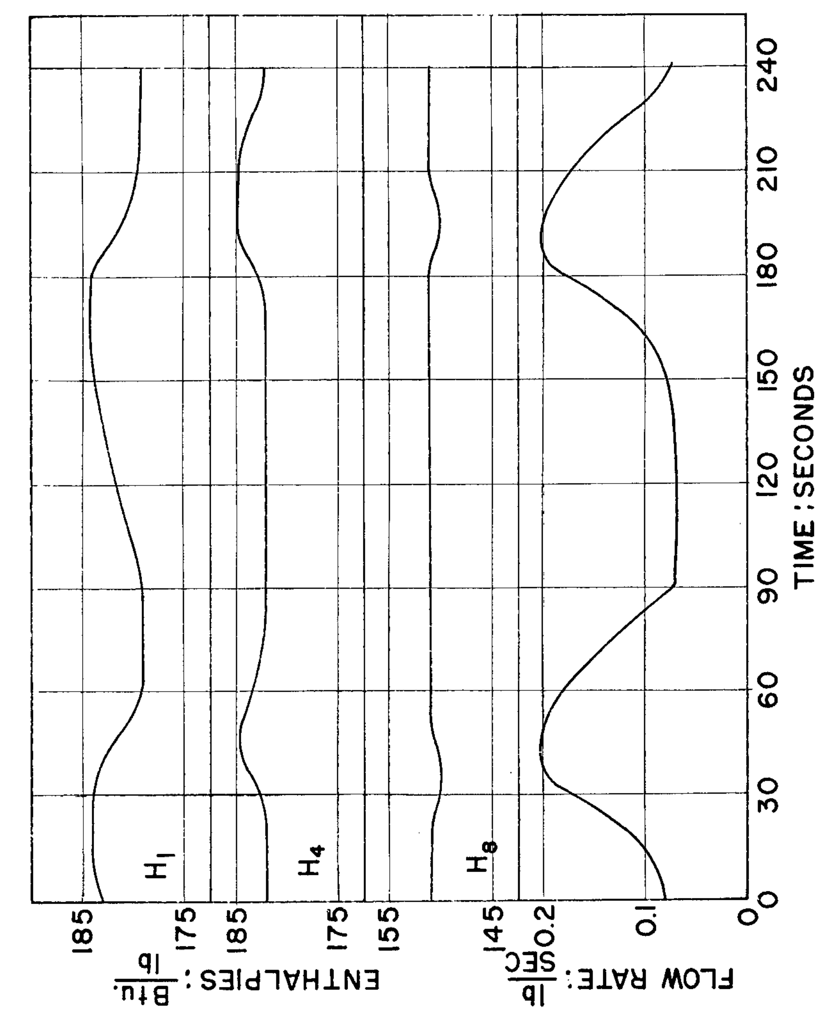

Figure 7. Computer of
the Analog Computer Problem.

Results

Two steady equilibrium modes of operation were possible when the pressure at one point in the loop is held constant (surge tank E_2 open to the atmosphere). For a very low heat input the water temperature in the riser never exceeds the boiling point and a state of stable equilibrium may be defined. For a very high heat input, the entire riser contains both steam and water, and a maximum flow rate is obtained. Oscillatory modes of operation result for the intermediate heat inputs. An illustration of the manner in which the period and amplitude depend on the heat input is given in Figures 8 to 12. The period and amplitude of the oscillations are determined by the mean temperature level of the fluid in the vertical riser.

The period was inversely proportional to the mean velocity providing some steam is in the riser at all times. When the system does not contain steam during most of the cycle, the period is considerably longer than that predicted by the extrapolation of the higher flow rate periods.

In the analogue computer problem, Q_1 = 3.16 Btu/sec, and the period was 149 seconds, which is about 21 seconds

less than the experimentally observed value for the lowest flow rate. The computed period would be expected to be less than the observed value, since the density function used in the top subinterval in the riser was a two-phase density function. Further, the density function used for the boiling subinterval was linear and it was not possible to exclude densities greater than the density of saturated water. A special non-linear element is required to generate a density function with the correct properties. As a result, the computed mean flow rate is less than that which could actually exist; however, the flow rate curve and enthalpy curves have essentially the same shape as the experimentally determined ones. A larger computer is required for solving the stability problem at the higher heat fluxes.

Conclusions

A natural circulation loop can be made unstable in the sense that a small displacement from the equilibrium state leads to undamped oscillations. Stable operations result when the fluid temperature in the riser is restricted to values less than the boiling point, and when the heat input reaches a value such that the frictional force

changes more rapidly than the driving force. The theoretical treatment of an open-ended natural circulation system and the solution of a simple problem using an analogue computer have lent support to the general conclusions of the stability analyses. A detailed numerical analyses is in progress on a large digital computer.

Acknowledgments

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NOTATION

Equations 1 through 4

A = cross-section area for flow

W = total mass flow rate

x = distance along streamline

t = time

g = local acceleration of gravity; g_c = conversion factor in Newton's law of motion

z' = slope of pipe at x

F_f = wall of frictional force

ρ = fluid density. $\rho \equiv \frac{1}{A}(A_w\rho_w + A_s\rho_s)$

V = volumetric flow rate. $V \equiv \frac{A_w\rho_w V_w + A_s\rho_s V_s}{A\rho} = \frac{W}{A\rho}$

H = specific fluid enthalpy. $H \equiv \frac{1}{A\rho}[A_w\rho_w H_w + A_s\rho_s H_s]$

$\phi_1 \equiv \frac{A_w\rho_w V_w^2 + A_s\rho_s V_s^2}{WV}$; $\phi_2 \equiv \frac{A_w\rho_w V_w H_w + A_s\rho_s V_s H_s}{WH}$

k = thermal conductivity

T = fluid temperature

p = fluid pressure

r = radius, r_p = pipe radius

Q' = rate at which heat is conducted through pipe.

$a(\rho)$ = an empirically defined function for Eq.4 for two-phase pressure drop.

Subscripts

s = steam phase; w = liquid water phase

<u>Equations 5 through 7</u>

The energy balance is reduced to a heat balance

H = specific enthalpy

Q = rate at which heat is added to fluid

V = volumetric flow rate

F = frictional force.

$$F = AV^{1.8} \cong F_o + \frac{1.8F_o}{V_o}(V-V_o) + \frac{0.72F_o}{V_o^2}(V-V_o)^2$$

F_o is the frictional force corresponding to a volumetric flow rate V_o.

h_c = cooler heat transfer coefficient

Δx = length of subinterval; z_n = vertical height of subinterval

ρ = density, $\bar{\rho}$ = mean density in a subinterval

C_p = heat capacity

α_n = a constant related to the coefficient of expansion of water and defined by Eq. 7

For the computer problem, for n = 1, 2, and 3, H_{n_o} = 182Btu/lb, ρ_{n_o} = 59.781 lb/cu.ft., α_n = - 0.024 lb/cu.ft.°F; n = 4,

$H_{n_o} = 182$, $\rho_{n_o} = 37.500$, $\alpha_n = -9.500$; $n = 5$ through 10,

$H_{n_o} = 153$, $\rho_{n_o} = 60.477$, $\alpha_n = -0.024$. $V_o = 0.0025$ cu.ft./sec,

$Q_1 = 3.16$ Btu/sec, and $Q_5 = -25.0V\left(\frac{H_5 + H_4}{2} - 89.6\right)$ Btu/sec.

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2. Isbin, H. S., Moen, R. H. and Mosher, D. R., AECU-2994, Nov. 1954.
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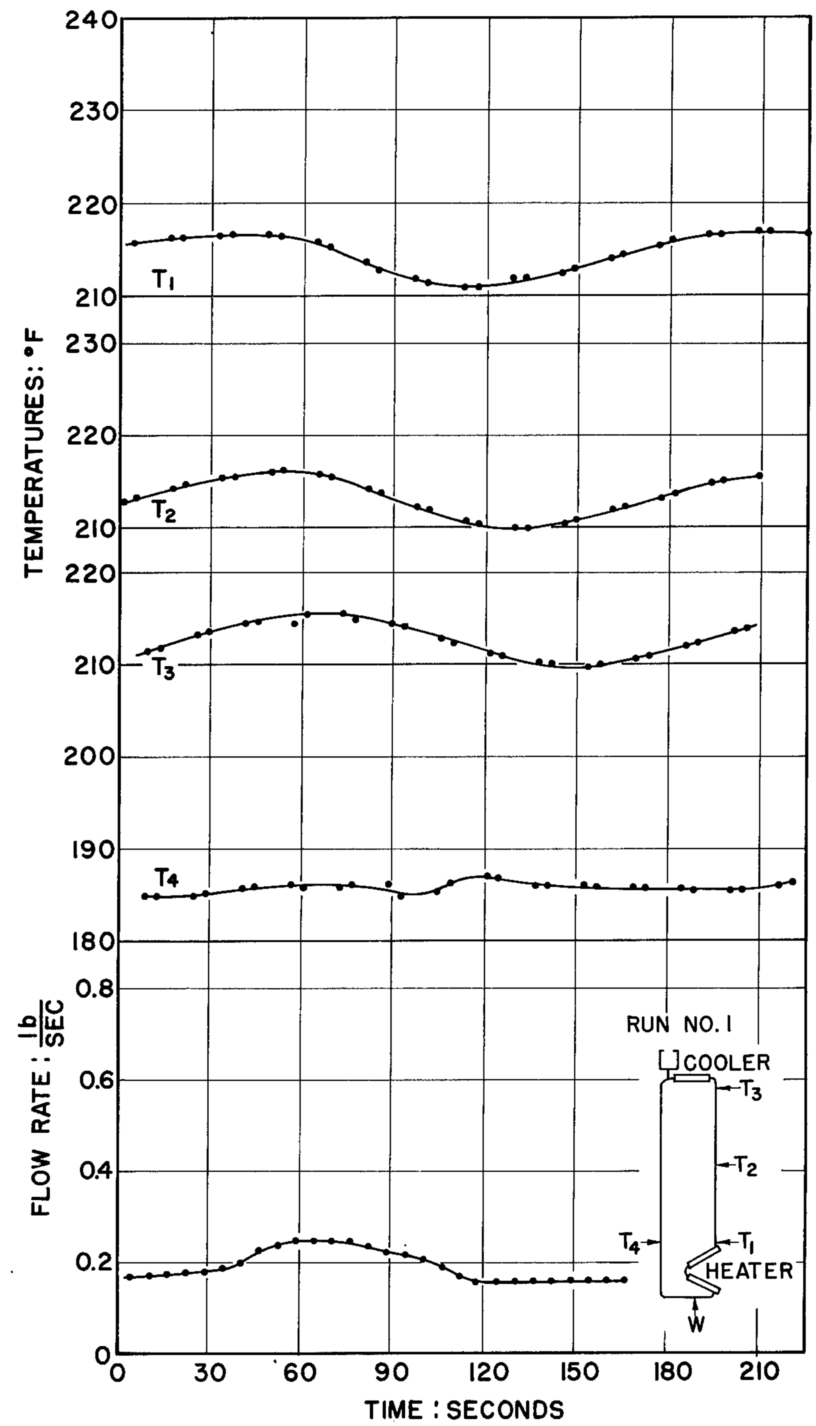
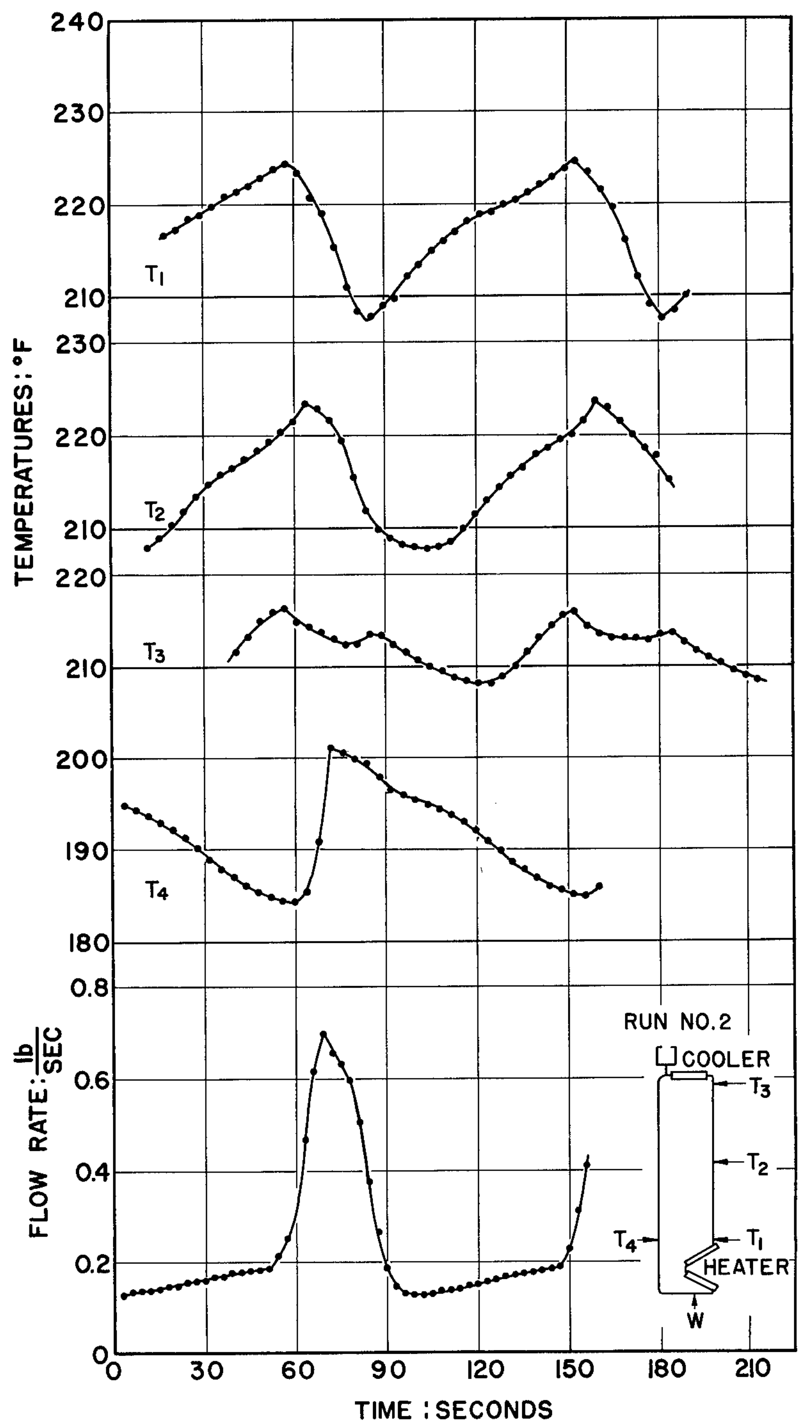


240
230
220
210
230
220
210
220
210
200
190
180
0.8
0.6
0.4
0.2
0
TEMPERATURES: °F
FLOW RATE: lb/SEC
T_1
T_2
T_3
T_4
RUN NO. 1
COOLER
HEATER
W
0
30
60
90
120
150
180
210
TIME : SECONDS



240
230
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230
220
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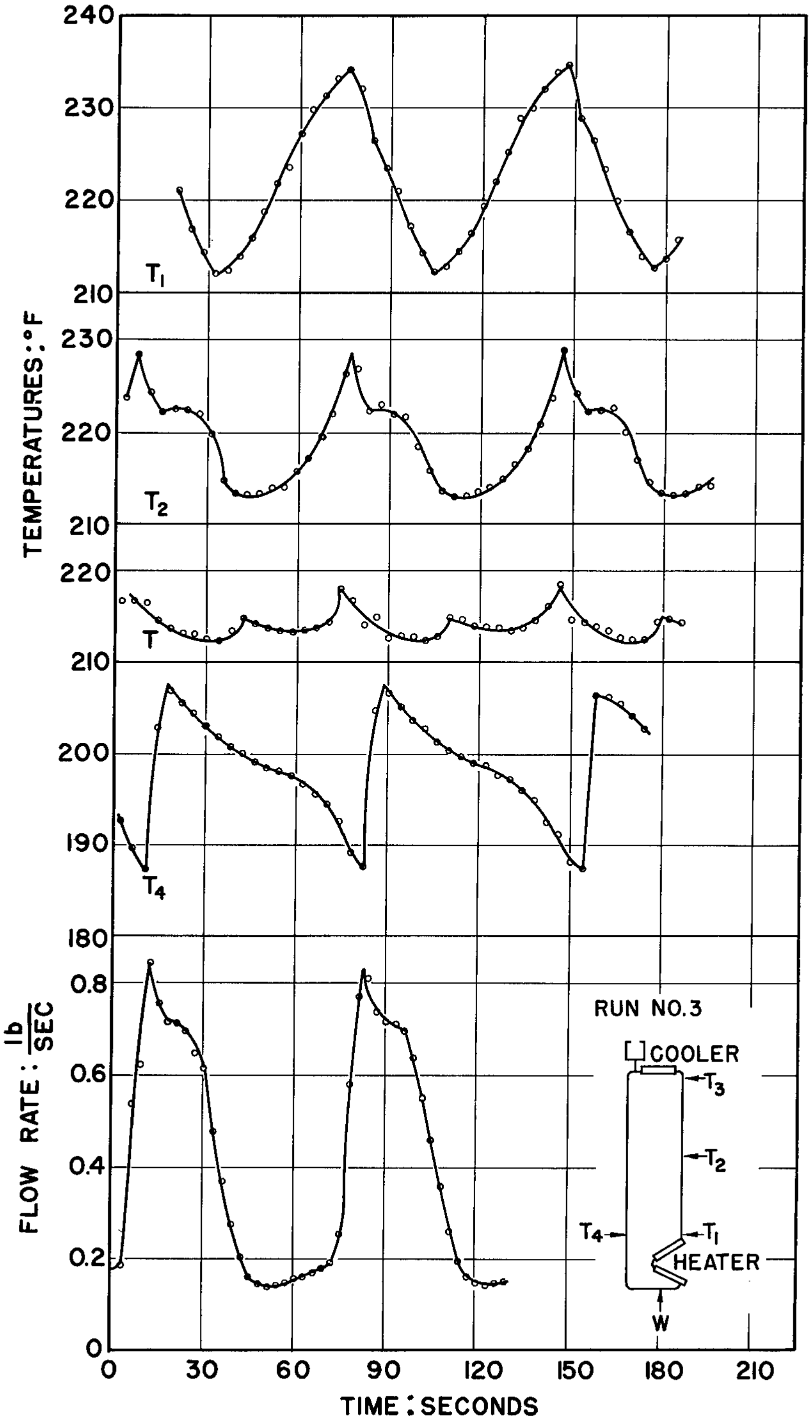
220
210
200
190
180
0.8
0.6
0.4
0.2
0
TEMPERATURES: °F
FLOW RATE: lb/SEC
T1
T2
T3
T4
RUN NO. 2
COOLER
HEATER
W
0
30
60
90
120
150
180
210
TIME : SECONDS



TEMPERATURES: °F
FLOW RATE: lb/SEC
TIME: SECONDS
T_1
T_2
T
T_4
RUN NO.3
COOLER
T_3
T_2
T_1
T_4
HEATER
W
240
230
220
210
230
220
210
220
210
200
190
180
0.8
0.6
0.4
0.2
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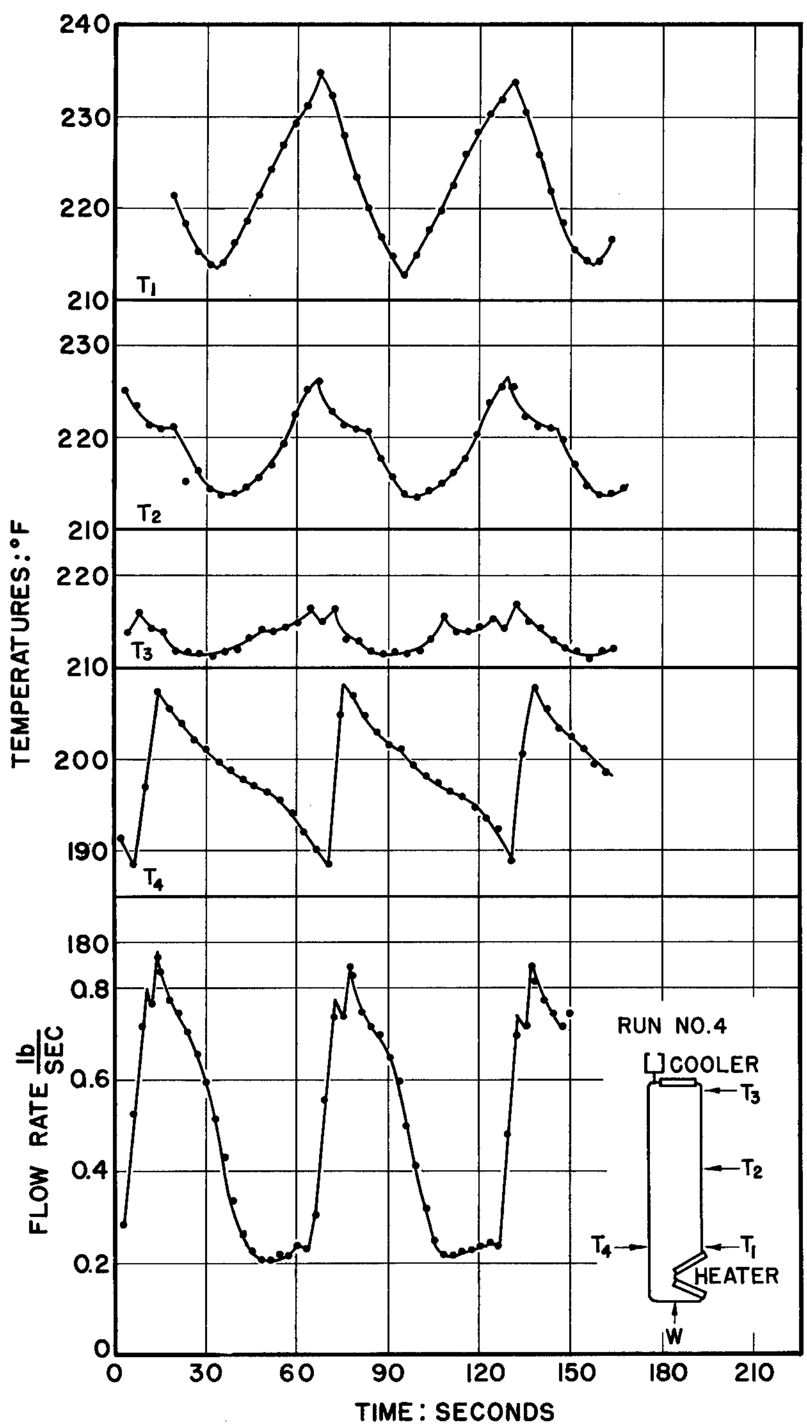
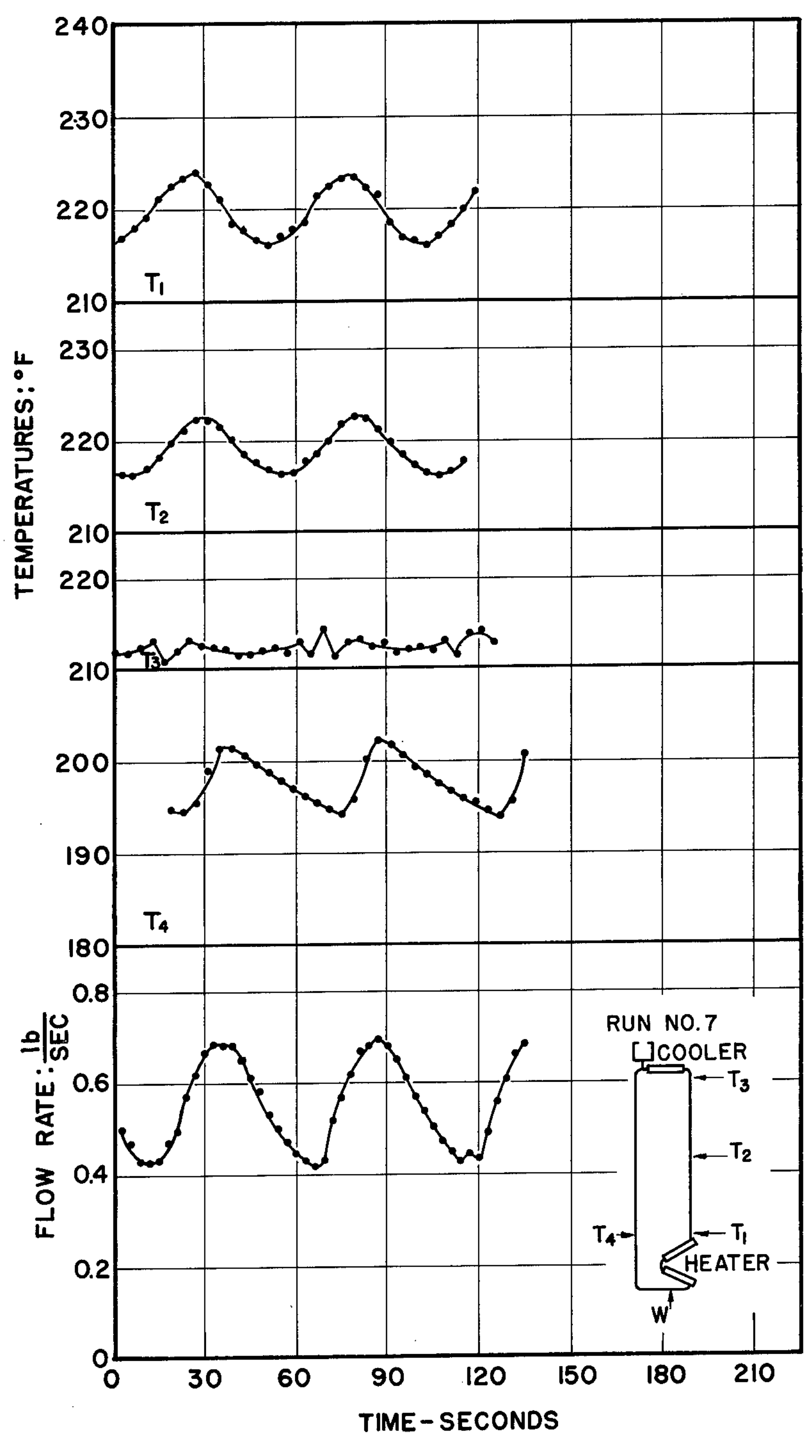


TEMPERATURES: °F
240
230
220
210
T1
230
220
210
T2
220
210
T3
200
190
T4
180
FLOW RATE lb/SEC
0.8
0.6
0.4
0.2
0
0
30
60
90
120
150
180
210
TIME: SECONDS
RUN NO.4
COOLER
T3
T2
T4
T1
HEATER
W


240
230
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T1
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T2
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T3
200
190
180
T4
TEMPERATURES: °F
0.8
0.6
0.4
0.2
0
FLOW RATE: lb/SEC
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30
60
90
120
150
180
210
TIME - SECONDS
RUN NO. 7
COOLER
T3
T2
T1
T4
HEATER
W